RAMAIAH
Pre-University College

II PUC Mid – term Examination- September 2018

Subject : MATHEMATICS **Time : 3hrs 15min**

Max marks : 100

Instructions: 1. The question paper has five parts namely A,B,C,D, and E. Answer all the five parts.

2. Use the graph sheet for the question on linear programming problem in part – E

PART – A

Answer all the ten questions **10×1 = 10**

1. Define Binary operation
2. Let $*$ be a binary operation on N ; given by $a*b$ = LCM(a,b). Find $20*16$
3. Write the principal value branch of $\cos^{-1}(x)$
4. Write the values of 'x' for which $2\tan^{-1}(x) = \cos^{-1}(\frac{1-x2}{1+x2})$ holds
5. Define a scalar matrix
6. If 'A' is an invertible matrix of order "2", then find A-1
7. If a is a square matrix of order 3, such that adj(A) = 64, find A
8. Differentiate $\sin\sqrt{x}$ with respect to 'x'
9. Find the derivative of log_{10}^{x} with respect to 'x'
10. Define feasible region

PART – B

Answer any ten questions **10×2 = 20**

11. Prove that the greatest integer function , f :R→R, defined by f(x) = [x],where,[x] indicates the greatest integer not greater than x, neither one-one nor onto
12. Let f :R→R be defined by $f(x) = x^2+1$. then find the pre-image of 17
13. Write the function $\tan^{-1}(\frac{cosx-sinx}{cosx+sinx})$ $0< x < \Pi$ in the simplest form
14. Evaluate : $\sin[\frac{\Pi}{3} - \sin^{-1}(\frac{-1}{2})]$
15. If $[2\ 3\ 5\ 7\ ][1\ -3\ -2\ 4\ ] = [-4\ 6\ -9\ x\ ]$ Write the value of x
16. If the area of the triangle with vertices (-2,0), (0,4), (0,k) is 4 sq. units. Find the values of 'K' using determinants
17. Find equation of the line joining (1,2) and (3,6) using determinants
18. Find $\frac{dy}{dx}$ if $x^2 + xy + y^2 = 100$

19. Find $\frac{dy}{dx}$ if y = $(\ logx)^{cosx}$

20. Verify Rolle's theorem for the function y = X^2 + 2 , x∈[-2,2]

21. Using differentials, find the approximate value of $(25)^{\frac{1}{3}}$

22. Find the slope of the tangent to the curve y = $\frac{x-1}{x-2}$; x≠ 2 at x = 10

PART – C

Answer any ten questions **10 × 3 = 30**

23. Check whether the relation 'R' in 'R' defines by R = { (a,b) ; a ≤ b^3} is reflexive , symmetric or transitive
24. Find g o f (x) and f o g (x) if f : R→ R and g : R → R are given by f(x) = cos x and g(x) = $3x^2$. Show that g o f (x) ≠ f o g(x)
25. Prove that , $\tan^{-1}(x) + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$
26. Solve for 'x' : $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\Pi}{3}$
27. By the elementary transformations, find the inverse of the matrix A = [$1\ -2\ 2\ 1$]
28. Find the values of x and y in [$x + 2y$ 2 4 $x + y$] - [3 2 4 1] = O, where O is a null matrix
29. Using the properties of determinants , prove that $1\ 1\ 1\ a\ b\ c\ bc\ ca\ ab$ = (a-b)(b-c)(c-a)
30. If x= sint ; y= cos(2t), then prove that, $\frac{dy}{dx}$= -4sint
31. Verify mean value theory if f(x) = $x^3 - 5x^2 - 3x$ in the interval [1,3]; xϵ [1,3]
32. If x = $\sqrt{a^{sin^{-1}}t}$ and y = $\sqrt{a^{cos^{-1}}t}$, then prove that $\frac{dy}{dx} = \frac{-y}{x}$
33. Find the intervals in which the function 'f' given by f(x) = $x^2 - 4x + 6$ is
    a. strictly increasing
    b. strictly decreasing
34. Find the two positive numbers whose sum is 15 and sum of whose squares is minimum

PART – D

Answer any six questions **6×5 = 30**

35. Let f: N→R be defined by f(x) = $4x^2 + 2x + 15$. Show that f : N→S, where 's' is the range of the function , is invertible . Also find the inverse of 'f'
36. If A = [1 0 2 0 2 1 2 0 3] ,Prove that, $A^3 - 6A^2 + 7A + 2I = 0$
37. If A = [$0\ 6\ 7\ -6\ 0\ 8\ 7\ -8\ 0$] B = [0 1 1 1 0 2 1 2 0] and C = $2\ -2\ 3$, Verify that, (A+B)C = AC + BC
38. Solve the following system of equations by matrix method ;

3x-2y+3z = 8 ; 2x+y–z = 1 ; 4x–3y +2z = 4

39. If y = $(\tan^{-1x})^2$. Show that $(x^2 + 1)^2 y_2$ + 2x(x^2 +1)y = 2
40. If y = Ae^{mx} + Be^{nx} , then show that, $\frac{d^2y}{dx^2}$ - (m+n) $\frac{dy}{dx}$ + mny = 0
41. The surface area of a spherical soap bubble increasing at the rate of 0.6 cm^2/ sec. Find the rate at which its volume is increasing when its radius is 3 cm.

PART – E

Answer all the questions **1×10 = 10**

42. a) Solve the following problem graphically.
Minimise and Maximise z = 3x+9y, subject to the constraints
$x+3y \leq 60$
$x+y \geq 10$
$x \leq y$
$x \geq 0, y \geq 0$ **(6)**
b) Find the relationship between 'a' and 'b', so that the function 'f' defined by
f(x) = $\{ ax + 1, if\ x \leq 3\ bx + 3, if\ x > 3$, is continuous at x = 3
(4)